CHAPTER 3 & 4

MATRICES AND DETERMINANTS

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

1. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y.

2. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ and $B = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, find AB.

3. Find the value of $a_{23} + a_{32}$ in the matrix $A = [a_{ij}]_{3 \times 3}$

   where $a_{ij} = \begin{cases} |2i - j| & \text{if } i > j \\ -i + 2j + 3 & \text{if } i \le j \end{cases}$.

4. If B be a 4×5 type matrix, then what is the number of elements in the third column.

5. If $A = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ find $3A - 2B$.

6. If $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -6 \end{bmatrix}$ find $(A+B)'$.

7. If $A = [1 \quad 0 \quad 4]$ and $B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ find AB.

8. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric matrix, then find x.

9. For what value of x the matrix $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -4 \\ 3 & 4 & x+5 \end{bmatrix}$ is skew symmetrix matrix.

10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = P + Q$ where P is symmetric and Q is skew-symmetric matrix, then find the matrix Q.

11. Find the value of $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

12. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$, find x.

13. For what value of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse.

14. If $A = \begin{bmatrix} \sin 30^\circ & \cos 30^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$, what is $|A|$.

15. Find the cofactor of a_{12} in $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

16. Find the minor of a_{23} in $\begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$.

17. Find the value of P, such that the matrix $\begin{bmatrix} -1 & 2 \\ 4 & P \end{bmatrix}$ is singular.

18. Find the value of x such that the points (0, 2), (1, x) and (3, 1) are collinear.

19. Area of a triangle with vertices (k, 0), (1, 1) and (0, 3) is 5 unit. Find the value (s) of k.

20. If A is a square matrix of order 3 and $|A| = -2$, find the value of $|-3A|$.

21. If $A = 2B$ where A and B are square matrices of order 3×3 and $|B| = 5$, what is $|A|$?

22. What is the number of all possible matrices of order 2×3 with each entry 0, 1 or 2.

23. Find the area of the triangle with vertices (0, 0), (6, 0) and (4, 3).

24. If $\begin{vmatrix} 2x & 4 \\ -1 & x \end{vmatrix} = \begin{vmatrix} 6 & -3 \\ 2 & 1 \end{vmatrix}$, find x.

25. If $A=\begin{bmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{bmatrix}$, write the value of det A.

26. If $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that $|A| = -15$, find $a_{11}C_{21} + a_{12}C_{22}$ where C_{ij} is cofactors of a_{ij} in $A = [a_{ij}]$.

27. If A is a non-singular matrix of order 3 and $|A| = -3$ find $|adj\ A|$.

28. If $A=\begin{bmatrix} 5 & -3 \\ 6 & 8 \end{bmatrix}$ find $(adj\ A)$

29. Given a square matrix A of order 3×3 such that $|A| = 12$ find the value of $|A\ adj\ A|$.

30. If A is a square matrix of order 3 such that $|adj\ A| = 8$ find $|A|$.

31. Let A be a non-singular square matrix of order 3×3 find $|adj\ A|$ if $|A| = 10$.

32. If $A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ find $\left|(A^{-1})^{-1}\right|$.

33. If $A=\begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ find $|AB|$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

34. Find x, y, z and w if $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3x+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

35. Construct a 3×3 matrix $A = [a_{ij}]$ whose elements are given by
$$a_{ij} = \begin{cases} 1+i+j & \text{if } i \geq j \\ \dfrac{|i-2j|}{2} & \text{if } i < j \end{cases}$$

36. Find A and B if $2A + 3B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 6 & 2 \end{bmatrix}$.

37. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)' = B'A'$.

38. Express the matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = P + Q$ where P is a symmetric and Q is a skew-symmetric matrix.

39. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ where n is a natural number.

40. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix D such that $CD - AB = O$.

41. Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

42. Prove that the product of the matrices

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and } \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is the null matrix, when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

43. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ show that $A^2 - 12A - I = 0$. Hence find A^{-1}.

44. If $A=\begin{bmatrix}2 & 3\\ 4 & 7\end{bmatrix}$ find $f(A)$ where $f(x) = x^2 - 5x - 2$.

45. If $A = \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$.

46. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$.

47. If $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$ then show that $(AB)^{-1} = B^{-1}A^{-1}$.

48. Test the consistency of the following system of equations by matrix method :

$$3x - y = 5;\ 6x - 2y = 3$$

49. Using elementary row transformations, find the inverse of the matrix $A=\begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}$, if possible.

50. By using elementary column transformation, find the inverse of $A=\begin{bmatrix}3 & 1\\ 5 & 2\end{bmatrix}$.

51. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and $A + A' = I$, then find the general value of α.

Using properties of determinants, prove the following : Q 52 to Q 59.

52. $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$

53. $\begin{vmatrix}x+2 & x+3 & x+2a\\ x+3 & x+4 & x+2b\\ x+4 & x+5 & x+2c\end{vmatrix}=0$ if a, b, c are in $A.P.$

54. $\begin{vmatrix}\sin\alpha & \cos\alpha & \sin(\alpha+\delta)\\ \sin\beta & \cos\beta & \sin(\beta+\delta)\\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta)\end{vmatrix}=0$

55. $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$

56. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$

57. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$

58. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c).$

59. Show that :

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy).$$

60. (i) If the points (a, b) (a', b') and $(a - a', b - b')$ are collinear. Show that $ab' = a'b$.

(ii) If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$ verity that $|AB| = |A||B|$.

61. Given $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$. Find the product AB and also find $(AB)^{-1}$.

62. Solve the following equation for x.

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$

63. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that,

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

64. Use matrix method to solve the following system of equations : $5x - 7y = 2$, $7x - 5y = 3$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

65. Obtain the inverse of the following matrix using elementary row operations

$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

66. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$x - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$.

67. Solve the following system of equations by matrix method, where $x \neq 0$, $y \neq 0$, $z \neq 0$

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10,\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10,\ \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13.$$

68. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, hence solve the system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

69. The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraically and find the numbers using matrix method.

70. Compute the inverse of the matrix.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 5 \end{bmatrix} \text{ and verify that } A^{-1} A = I_3.$$

71. If the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then compute $(AB)^{-1}$.

72. Using matrix method, solve the following system of linear equations :

$$2x - y = 4,\ 2y + z = 5,\ z + 2x = 7.$$

73. Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Also show that $A^{-1} = \dfrac{A^2 - 3I}{2}$.

74. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary column transformations.

75. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5.

76. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A \cdot (adj\ A) = (adj\ A) \cdot A = |A|\ I_3$.

77. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$, hence find A^{-1}.

78. Find the matrix X for which

$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} . X . \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$

79. By using properties of determinants prove the following :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

80. $$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3.$$

81. $$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$$

82. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$. Show that $xyz = -1$.

83. If x, y, z are the 10^{th}, 13^{th} and 15^{th} terms of a G.P. find the value of

$$\Delta = \begin{vmatrix} \log x & 10 & 1 \\ \log y & 13 & 1 \\ \log z & 15 & 1 \end{vmatrix}.$$

84. Using the properties of determinants, show that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$$

85. Using properties of determinants prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

86. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$3x + 4y + 7z = 14,\ 2x - y + 3z = 4,\ x + 2y - 3z = 0.$

ANSWERS

1. $x = 2,\ y = 7$

2. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

3. 11.

4. 4

5. $\begin{bmatrix} 9 & -6 \\ 0 & 29 \end{bmatrix}$.

6. $\begin{bmatrix} 3 & -5 \\ -3 & -1 \end{bmatrix}$.

7. $AB = [26]$.

8. $x = 5$

9. $x = -5$

10. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

11. $a^2 + b^2 + c^2 + d^2$.

12. $x = -13$

13. $k = \frac{3}{2}$

14. $|A| = 1$.

15. 46

16. –4

17. $P = -8$

18. $x = \frac{5}{3}$.

19. $k = \frac{10}{3}$.

20. 54.

21. 40.

22. 729

23. 9 sq. units

24. $x = \pm 2$

25. 0

26. 0

27. 9

28. $\begin{bmatrix} 8 & 3 \\ -6 & 5 \end{bmatrix}$.

29. 1728

30. $|A| = 9$

31. 100

32. 11

33. $|AB| = -11$

34. $x = 1,\ y = 2,\ z = 3,\ w = 4$

35. $\begin{bmatrix} 3 & 3/2 & 5/2 \\ 4 & 5 & 2 \\ 5 & 6 & 7 \end{bmatrix}$.

36. $A = \begin{bmatrix} \frac{11}{7} & -\frac{9}{7} & \frac{9}{7} \\ \frac{1}{7} & \frac{18}{7} & \frac{4}{7} \end{bmatrix}$, $B = \begin{bmatrix} -\frac{5}{7} & -\frac{2}{7} & \frac{1}{7} \\ \frac{4}{7} & -\frac{12}{7} & -\frac{5}{7} \end{bmatrix}$

40. $D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$.

41. $x = -2$ or -14

43. $A^{-1} = \begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$.

44. $f(A) = 0$

45. $x = 9,\ y = 14$

46. $x = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

48. Inconsistent

49. Inverse does not exist.

50. $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$.

51. $\alpha = 2n\pi \pm \frac{\pi}{3},\ n \in z$

61. $AB = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$, $(AB)^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}$.

62 0, $3a$

64. $x = \frac{11}{24},\ y = \frac{1}{24}$.

65. $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$.

66. $x = 0,\ y = 5,\ z = 3$

67. $x = \frac{1}{2},\ y = \frac{1}{3},\ z = \frac{1}{5}$

68. $A^{-1} = -\frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

69. $x = 1,\ y = -2,\ z = 2$

70. $A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

71. $(AB)^{-1} = \frac{1}{19}\begin{bmatrix} 16 & 12 & 1 \\ 21 & 11 & -7 \\ 10 & -2 & 3 \end{bmatrix}$.

72. $x = 3,\ y = 2,\ z = 1$.

73. $A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

74. $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

75. $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$.

77. $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$.

78. $X = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$.

83. 0

86. $x = 1,\ y = 1,\ z = 1$.